

## Mixing Patterns in Worldwide Earthquake Networks

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### Abstract

In this work, we have studied the assortativity of worldwide earthquake networks. We have used data from the worldwide earthquake catalog for the period between 2002 and 2016 of earthquakes with magnitude  $m \geq 4.5$ . The study was conducted for two different networks: the first one using data of shallow earthquakes (depth up to 70km) and the second constructed for deep events (depth greater than 70km), both for the world. We calculated the average nearest neighbors' degree of nodes (ANND) distribution and the degree correlation coefficient (DCC) for both networks. It was observed that the network of shallow events is assortative. The same result was found in previous studies for networks of earthquakes from California and Japan, and also for networks of earthquakes produced by computer simulations. The network of deep earthquakes was found to be neutral. Our results contribute to the understanding of the seismological dynamics and features.

**Key Words:** Earthquakes; Complex networks; Degree correlation; Assortativity;

### Introduction

In the last years, several works have implemented the concept of complexity in the study of earthquakes, aiming to better understand and characterize the seismological dynamics and properties. Complex networks are a powerful tool for investigating the topological structure and statistical properties of complex systems and have been applied in many real-world networks, such as economic market, internet, spread of diseases, solar flares and social relationships [1-7].

Some of the most common features of networks, that have fundamental implications on the networks dynamics, are the clustering coefficient, the average shortest path and the degree distribution. However, another crucial characteristic to be studied is the networks' assortativity, which shows the likelihood of a node with degree  $k$  to be connected to other nodes of the same degree  $k$ . For example, in social networks it is observed that people tend to relate to other people belonging to the same group as themselves [7] and for this reason this network is assortative. However, the

protein-interaction network of yeast has the opposite property: it is disassortative [8]. The proteins with larger values of degree interacts much more with small-degree proteins. This way, the study of degree correlation is relevant, since it can describe the robustness of a network against selective node failure [9,10].

In [11,12], the authors created a model where they have constructed an earthquake network using a successive model for creating the links between the nodes for seismic data from California and Japan. They found that these networks were small-world and scale-free. Moreover, the same authors analyzed these networks in respect to their assortative mixing [13], and found that they are assortative. This characteristic was also found for the network using earthquake data produced by the Olami-Feder-Christensen model (known as the OFC model) [14]. Intending to study earthquakes from the whole world and not just of a small region, in [15] it was constructed a complex network of global earthquakes, where the authors found that this network was also scale-free and small-world, showing evidence of long-range correlations across the planet.

In order to corroborate the understanding of the phenomenon of earthquakes and to observe the agreement between the results found for specific regions of the world and in a global perspective, in this paper we analyze the assortativity of the network created for worldwide earthquakes using the successive model of connections for two datasets: one network of shallow earthquakes and one network of deep earthquakes. This division aims to make comparisons between earthquakes with similar seismic origins.

This paper is organized as follows. First, we present information about assortative mixing, the degree correlation coefficient (DCC) and the average nearest neighbors' degree of nodes (ANND). In the sequence, we give information about our worldwide earthquake data, as well as the method employed to construct our networks. We then calculate the DCC and ANND for the networks and discuss the results. Finally, we end with the conclusions.

### Assortative mixing in networks

The study of assortativity in networks has been implemented in many different real-world networks in the last years. The analysis of this property is an important tool to investigate the preference of a  $k$ -degree node to be connected to another node that has degree  $k$ . When it happens, the network is said to have assortative mixing. When the preference is the opposite, i.e. the nodes prefer to be connected to others with different degree value, then this network is disassortative. If the nodes do not have a preference of connection, the network is classified as neutral [9].

Two statistical measures that are commonly used to analyze this preference are the average nearest neighbors' degree of nodes (ANND) [16,17] and the degree correlation coefficient (DCC) [7,9]. The ANND or the degree correlation function is expressed a

$$k_{nn}(k) = \sum_j jP(j|k), \quad (1)$$

where  $P(j|k)$  is the conditional probability that an arbitrary selected edge links a  $j$ -degree node with a  $k$ -degree node. This function considers the average degree of the neighbors of a node as a function of its degree  $k$ . If it is independent of  $k$ , the network has no obvious correlation of degree. As reported in [17], it is possible to approximate the ANND to

$$k_{nn}(k) = ak^\mu \quad (2)$$

where the sign of the correlation exponent  $\mu$  determines the behavior of degree correlation. For assortative networks, the correlation exponent is positive ( $\mu > 0$ ) and for disassortative networks it is negative ( $\mu < 0$ ). When  $\mu = 0$ ,  $k_{nn}(k)$  presents no dependence with  $k$ .

The DCC (which is the Pearson correlation coefficient between the degrees found at the two end of the same link) is a complementation of the analysis of degree correlation and gives to us a quantitative characterization. With this coefficient it is possible

to know if the network is assortative, disassortative or neutral, but also represents the strength of this correlation. We calculate this coefficient by

$$r = \sum_{jk} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2}, \quad (3)$$

where  $q_k$  is the probability of existing a node with degree  $k$  at the end of a randomly selected edge and

$$\sigma^2 = \sum_k k^2 q_k - \left[ \sum_k k q_k \right]^2 \quad (4)$$

is the variance of  $q_k$ . The value of  $r$  varies from -1 (perfect disassortativity) to 1 (perfect assortativity). If  $r = 0$ , then the network has no assortative (or disassortative) mixing and, therefore, is neutral.

### Method

Following the definition used in [15] for the vertices of the worldwide epicenters network, we divide the planet into equal square cells of size  $L \times L$ , with  $L = 20\text{km}$ , and a cell becomes a vertex of the network every time the epicenter of an earthquake is located therein. To create the links between the vertices, we used the successive model employed in [11,12,15]. It basically consists of connecting a vertex to its subsequent one in the temporal order by a directed edge.

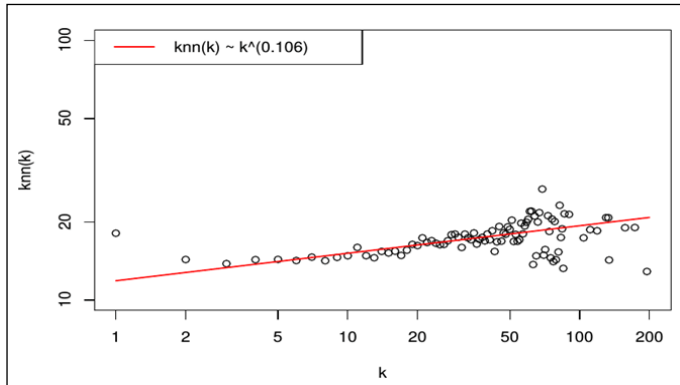
The dataset used in our study was obtained from the World have of Earthquakes of Advanced National Seismic System (ANSS)<sup>b</sup> (<https://earthquake.usgs.gov/data/comcat/>) and it covers earthquakes from the entire world between 2002 and 2016. For the record, we only considered earthquakes with magnitude ( $m$ ) larger or equal to 4.5, because in that catalog the events with magnitudes less than 4.5 are not completely registered for the whole world. The total of events is 101746, where 80520 are shallow earthquakes (earthquakes with depth up to 70km) and 21226 are deep earthquakes (events occurred at depths greater than 70 km).

We built a network for each of these two datasets and applied the equations exposed in the previous section. The results obtained is shown in the next section.

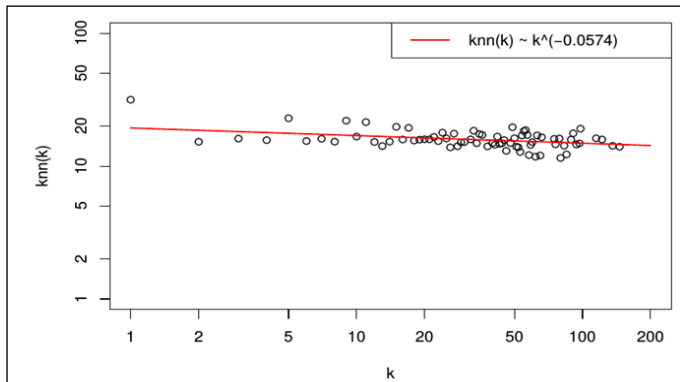
### Results and Discussion

From equation (1), we constructed the ANND distribution, using the in-degree of the vertices, for our networks. In Figure. 1, we can observe that the degree correlation function of the network of shallow earthquakes has an increasing trend, i.e.,  $k_{nn}(k)$  increases with  $k$ , which means that vertices with larger (lower) connections tend to connect with vertices with larger (lower) numbers of connections. It means that this network is assortative, as it was found for networks using the Olami-Feder-Christensen model [14] and for networks of earthquakes from California and Japan [13]. It is worth mentioning that most earthquakes that occur in Japan and

California have depth up to 70km (shallow earthquakes). Figure 2 presents the result for the network of deep earthquakes, where the correlation exponent is  $\mu \sim 0$ , meaning that this network is neutral.



**Figure 1:** Average nearest neighbors' degree of nodes  $k_{nn}(k)$  (for in-degree) for the network of shallow earthquake. The correlation exponent is positive ( $\mu > 0$ ), which means that this network is assortative.



**Figure 2:** The average nearest neighbors' degree of nodes  $k_{nn}(k)$  (for in-degree) for the network of deep earthquakes. No degree correlation between the nodes was found, since  $\mu \sim 0$ . This network is neutral.

The degree correlation coefficient (DCC) was calculated for each of our networks and the values are shown in Table 1. As it can be seen, the network of shallow earthquakes presents a positive value for the DCC ( $r = 0.193$ ), while the network of deep earthquakes has a value  $r \sim 0$ . Again, the values obtained for the degree correlation coefficient results in an assortative mixing in the shallow global earthquakes network and no degree correlation in the deep global earthquakes network.

**Table 1:** The number of nodes, the values of the correlation exponent ( $\mu$ ) and the values of the Pearson correlation coefficient ( $r$ ) of the networks of global earthquakes for the two datasets used in this work.

Network	N	$\mu$	$r$
Shallow earthquakes	28471	0.106	0.193
Deep earthquakes	8958	-0.0574	0.008

## Conclusion

In this work, the assortativity of networks of worldwide earthquakes is studied for two datasets: one for shallow earthquakes and other for deep earthquakes. To do this, these complex networks were constructed using the successive model of connections between the vertices of the network and, then, two analyzes of the degree correlation were done: the average nearest neighbors' degree of nodes (ANND) and the degree correlation coefficient (DCC). It was obtained that the network of shallow earthquakes is assortative, similarly to the results found for networks constructed with data from California and Japan (regions with predominance of shallow earthquakes) and also for networks created using catalogs produced by the computational model proposed by Olami, Feder and Christensen (OFC model). On the other hand, the deep earthquakes network presented no correlation between the degree of the vertices.

For future works, we plan to make these analyzes for global earthquake networks created with an improved methodology of linking the vertices of the networks to observe whether the results of assortative network for the shallow data and of neutral network for the deep data are maintained. The results will be published elsewhere.

## Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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